Rayat Shikshan Sanstha's

Karmaveer Bhaurao Patil College Vashi, Navi Mumbai

Autonomous College

[University of Mumbai]

Syllabus for Approval

Sr. No.	Heading	Particulars
1	Title of Course	F.Y.B.Sc. Mathematics
2	Eligibility for Admission	12th Science and equivalent [of recognized Boards]
3	Passing Marks	40%
4	Ordinances/Regulations (if any)	
5	No. of Years/Semesters	One year/Two semester
6	Level	U.G.
7	Pattern	Semester
8	Status	New
9	To be implemented from Academic year	2018-2019

Date: ___________________

Signature: _____________________

Name of BOS Chairman: ____

AC- 01/09/ 2018

Item No-2.26

Rayat Shikshan Sanstha's

KARMAVEER BHAURAO PATIL COLLEGE, VASHI.
NAVI MUMBAI

Sector-15- A, Vashi, Navi Mumbai - 400 703

(AUTONOMOUS COLLEGE)

Syllabus for Mathematics

Program: B.Sc.

Course: F.Y.B.Sc. Mathematics

(Choice Based Credit, Grading and Semester System with effect from the academic year 2018-2019)

Preamble of the Syllabus:

Bachelor of Science (B.Sc.) in Mathematics is a under graduation programme of Department of Mathematics, Karmaveer Bhaurao Patil College Vashi, Navi Mumbai [Autonomous College]

The Choice Based Credit and Grading System to be implemented through this curriculum would allow students to develop a strong footing in the fundamentals and specialize in the disciplines of his/her liking and abilities. The students pursuing this course would have to develop understanding of various aspects of the mathematics. The conceptual understanding, development of experimental skills, developing the aptitude for academic and professional skills, acquiring basic concepts and understanding of hyphenated techniques are among such important aspects.

Rayat Shikshan Sanstha's
KARMAVEER BHAURAO PATIL COLLEGE, VASHI
[AUTONOMOUS COLLEGE]

Department of Mathematics
B. Sc. Mathematics

Program Specific Outcomes

A. Knowledge and understanding

On completion of this Programme the student will have knowledge and understanding of:

1. Core areas of pure mathematics including geometry, algebra, real analysis and Topology of metric spaces.
2. Core areas of applied mathematics including ordinary differential equations and discrete mathematics.
3. The correct use of mathematical language to express both theoretical concepts and logical argument.
4. The use of computers both as an aid and as a tool to study problems in mathematics.
5. Think in a critical manner.
6. Acquire good knowledge and understanding in advanced areas of mathematics, chosen by the student from the given courses.

B. Cognitive (thinking) skills

On completion of this Programme the student will be able to:

1. Know when there is a need for information, to be able to identify, locate;
2. Formulate problems in appropriate theoretical frameworks to facilitate their solution;
3. Develop strategies to solve mathematical problems in a range of relevant areas;
4. Construct logical arguments for solving abstract or applied mathematical problems.

C. Practical skills

On completion of the Programme the student will be able to:

1. Solve practical problems in a range of areas of mathematics;
2. Determine the appropriateness of different methods of solving mathematical problems;
3. Communicate mathematics effectively to a wide range of audiences;
4. Use computer packages where appropriate to develop a deeper understanding of mathematical problems.

D. Graduate skills

On completion of this Programme the student will be able to:

1. Work effectively and constructively as part of a team;
2. Motivate and communicate complex ideas accurately using a range of formats;
3. Identify and benefit from opportunities for personal and career development;
4. Work confidently and accurately with formulae and numerical information
5. Learn effectively.

Syllabus

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Semester I

Calculus I				
Course Code	Unit	Topics	Credits	L/Week
UGMT101	I	Real Number System	2	3
	II	Sequences		
	III	Series		
Algebra I				
UGMT102	I	Introduction to logic and functions	2	3
	II	Integers and polynomials		
	III	Prime numbers and Polynomials		
Practical				
UGMTP01		Practical Based on UGMT101 and UGMT102	2	2

Semester II

Calculus II				
Course Code	Unit	Topics	Credits	L/Week
UGMT201	I	Limit and continuity of function	2	3
	II	Continuous functions and Differentiation		
	III	Application of Differentiation		
Algebra II				
UGMT202	I	System of linear equations and Matrices	2	3
	II	Vector spaces		
	III	Basis		
Practical				
UGMTP02		Practical Based on UGMT201 and UGMT202	2	2

Teaching Pattern

1. For UGMT101, UGMT102, UGMT201 and UGMT202, three lectures per week per course. Each lecture should be of 48Minutes duration.
2. For UGMTP01 and UGMTP02, one practical per week per batch. Each practical should be of 2 lectures.

SEMESTER I
UGMT101: CALCULUS I

Unit I: Real Number System (15 Lectures)
Learning Outcomes:

1. State the order properties and properties of real numbers.
2. Define neighborhood of a point.
3. Define arithmetic mean and geometric mean of given set of real numbers and illustrate the Arithmetic mean- Geometric mean inequality.
4. Apply arithmetic and geometric mean inequality to prove some inequalities.
5. Apply Hausdroff property to find disjoint neighborhood of two distinct real numbers.
6. State Archimedean property.

Content of the Unit:

Real number system and order properties of $\mathbb{R}$, Absolute value properties, AM-GM inequality, Triangle inequality, Intervals and neighborhood, Hausdorff property, Bounded sets, Continuum property, l.u.b. and g.l.b axiom statement and its consequences, Density of rational and irrational, Nested interval theorem, Archimedean property and its applications.

Unit II: Sequences (15 Lectures)

Learning Outcomes:

1. Define a sequence and classify different types of sequence.
2. Discuss the behavior of the geometric sequence and series.
3. State the properties of convergent and divergent sequences.
4. Give examples for convergence, divergence and oscillating sequence.
5. Verify the given sequence in convergent and divergent by using behavior of Monotonic sequence.
6. Explain subsequences and upper and lower limits of a sequence.

Content of the Unit:

Definition of a sequence and examples, Convergence of sequence, every convergent sequence is bounded, uniqueness of limit if exists, Convergence of standard sequences, Algebra of convergent sequences, Sandwich theorem, Cauchy sequence, Bolzano Weierstrass theorem, Monotone sequences, Monotone convergence theorem, Subsequences definition, Subsequence of a convergent sequence is convergent and converges to the same limit, every convergent sequence is a Cauchy sequence and converse.

Unit III: Series (15 Lectures)
Learning Outcomes:

1. Define series of real numbers.
2. Express series as a sequence of partial sums.
3. Give examples for convergence, divergence and oscillating series.
4. Verify the given series is convergent or divergent by using different tests of convergence of series.
5. Find the sum of a convergent series.

Content of the Unit:

Series of real numbers, simple examples of series, series as a sequence of partial sums, convergence of series, convergent and divergent series, Necessary condition for convergence of series, Algebra of convergent series, Cauchy criterion, Comparison test, limit comparison test, Geometric series, Telescopic series Alternating series, Leibnitz theorem (alternating series test), Absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test, root test and examples.

Reference Books:

1. Ajit Kumar-S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014

2. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

Additional Reference Books:

1. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pvt. Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

Assignments:

1. Application based examples of Archimedean property, intervals and neighborhood.
2. Consequences of l.u.b. and g.l.b. axiom, infimum and supremum of sets.
3. Calculating limits of sequences.
4. Cauchy sequences, monotone sequences.
5. Calculating limit of series, Convergence tests.
6. l.u.b and g.l.b. of different sets.

UGMT102: ALGEBRA I

Unit I. Introduction to logic and Functions (15 Lectures)
Learning Outcomes:

1. Explain statements and logic and various methods of proof.
2. Define a set and explain the basic concept of set theory such as union, intersection and complement.
3. Define relations, equivalence relations and determine if a relation is an equivalence relation and find the corresponding equivalence class.
4. Define functions and classify different types of functions.
5. Determine whether a given function is Bijective and hence invertible.

Content of the Unit:

Statements and logic, Methods of proof, examples, and Basic Set theory: Union, intersection and complement, indexed sets, the algebra of sets, power set, Cartesian product, relations, equivalence relations, partitions, discussion of the example congruence modulo-m relation on the set of integers.
Definition of function, domain, co-domain and range of a function, images and inverse images of sets under function composite functions, graph of the function, Types of functions & their properties, Bijective functions are invertible and conversely, Examples of functions, Binary operation as a function.

Unit II: Integers and polynomials (15 Lectures)
Learning Outcomes:

1. State Well-ordering property of integers.
2. Construct Pascal's Triangle.
3. State Binomial theorem for non-negative exponents and apply it to find coefficients of terms in the expansion.
4. Compute Least common multiple and Greatest common divisor of two integers.
5. Compute Greatest Common Divisor of two polynomials.

Content of the Unit:

Statement of well-ordering property of non-negative integers, induction principle as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers. Definition of polynomial, Algebra of polynomials, degree of polynomial, basic properties, Division algorithm in polynomials and g.l.b. of two polynomials and its basic properties, Euclidean algorithm applications.

Unit III: Prime numbers and Polynomials (15 Lectures)
Learning Outcomes:
1. Define prime numbers. State Euclid's Lemma.
2. State the Fundamental Theorem of arithmetic.
3. Apply Fermat's theorem and Wilson's theorem to find the remainder when an integer is divided by a prime number.
4. Define congruence modulo relation and state its properties.
5. Explain the various properties of integers and algebra of polynomials and determine the roots of a given polynomial and vice-versa.

Content of the Unit:

Primes, Euclid's lemma, Fundamental theorem of arithmetic, the set of primes are infinite. Definition and elementary properties of congruence, Euler's phi function, Statements of Euler's theorem, Fermat's little theorem, Wilson theorem and applications.
Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, a polynomial of degree n has at most n roots. Complex roots of a polynomial occur in conjugate pairs, Statement of Fundamental Theorem of Algebra.

Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.
3. Foundation Course in Mathematics by Ajit Kumar, S. Kumaresan and Bhaba Sarma Narosa Publication 2017.
4. Robert R. Stoll: Set theory and logic, Freeman & Co.

Additional Reference Books:

1. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.
2. Elementary number theory, Jones and Jones.
3. Number theory and its applications, Thomas Koshy.

Assignments:
1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2. Division Algorithm and Euclidean algorithm in Z, primes and the Fundamental Theorem of Arithmetic.
3. Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions.
4. Congruences and Euler's-function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5. Equivalence relation.
6. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

Practical based on UGMT101:
1. Application based examples of Archimedean property, intervals and neighborhood.
2. Consequences of l.u.b. & g.l.b. axiom, infimum and supremum of sets.
3. Calculating limits of sequences. Cauchy sequences, monotone sequences.
4. Calculating limit of series
5. Convergence tests.

Practical based on UGMT102:
1. Mathematical induction
2. Division Algorithm and Euclidean algorithm in $\mathbb{Z}$, primes and the Fundamental Theorem of Arithmetic.

3. Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions.
4. Congruence and Euler'sφ-function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5. Equivalence relation. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

SEMESTER II
UGMT201: CALCULUS II

Unit I: Limit and continuity of function
Learning Outcomes:

1. Define continuity and sequential continuity and limits of real valued functions at a point and interval.
2. Define discontinuous functions and classify the discontinuity as removable discontinuity or essential discontinuity.
3. Determine whether the given function is continuous using $\varepsilon - \delta$ definition of continuity.
4. State and prove algebra of limits, continuous functions and differentiability.
5. State and prove properties of continuous functions.

Content of the Unit:

Continuity of a real valued function on a set, $\in -\delta$ definition, examples, Continuity of a real valued function at end points of a domain, Sequential continuity, $\in -\delta$ if and only if Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity.
Graphs of some standard functions, Limit of a function, evaluation of limit of simple functions using $\in -\delta$ definition, uniqueness of limit if it exists, Algebra of limits Limit of composite function, Sandwich theorem, Left hand, right hand limits, non existence of limits, limit as t$\rightarrow \pm\infty$.

Unit II: Continuous functions and Differentiation (15 Lectures)
Learning Outcomes:

1. Define differentiation at a point and an open set and at the end points of an interval.
2. Apply chain rule to find derivative of composite functions.
3. Compute left hand derivative and right hand derivative and determine whether a given function is differentiable at a given point.
4. Find the derivative of implicit functions.

Content of the Unit:

Properties of Continuous functions: $f([a,b])$ is closed if f is continuous, Intermediate value theorem and its applications, Bolzano-Weierstrass theorem. A continuous function on a closed and bounded interval is bounded and attains its bounds.
Differentiation of real valued function of one variable: Definition of differentiation at a point and on an open set, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, Algebra of differentiable functions, chain rule, Derivative of inverse functions, Implicit differentiation.

Unit III: Application of differentiation (15 Lectures)
Learning Outcomes:

1. Determine local maxima, local minima and stationary points using second derivative test.
2. Apply L-Hospital rule to find the limits of indeterminate forms.
3. Define higher order derivatives and various methods to find higher order derivatives.

Content of the Unit:

Definition of local maximum and local minimum, necessary conditions, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, points of inflection, Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Higher order derivatives, Leibnitz rule, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications, L-hospital rule without proof, examples of indeterminate forms.

Reference Books:

1. R.G. Bartle- D.R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.
2. Ajit Kumar- S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

Additional Reference Books:

1. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
2. T. M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Private. Ltd.
3. Ghorpade, Sudhir R.- Limaye, Balmohan V., A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

Assignments:

1. Limit of a function and Sandwich theorem.
2. Continuous and discontinuous functions.
3. Properties of continuous functions.
4. Differentiability, Higher order derivatives, Leibnitz theorem.
5. Mean value theorems and its applications.
6. Extreme values, increasing and decreasing functions
7. Applications of Taylor's theorem and Taylor's polynomials.

UGMT202: ALGEBRA II

Unit I: System of Linear equations and Matrices (15 Lectures)
Learning Outcomes:

1. Formulate the parametric equation of lines and planes.
2. Define matrices, types of matrices and invertible matrices.
3. Perform elementary row operations to reduce a matrix to its row echelon form.
4. Define homogenous and non-homogeneous system of linear equations.
5. Express system of linear equations in matrix form.
6. Solve an exercise on system of linear equations and find the solution to a system if it exists.

Content of the Unit:

Parametric equation of lines and planes, System of homogeneous and non homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation, Matrices with real entries, addition, scalar multiplication and multiplication of matrices, Transpose of a matrix, Types of matrices, Invertible matrices, identities. System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m "homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector spaces (15 Lectures)
Learning Outcomes:

1. Define Vector Space, subspace over a field $\mathbb{R}$.
2. Determine whether a given set is a vector space over $\mathbb{R}$.
3. Apply subspace test to find whether a given set is a subspace of the vector space.
4. Explain the properties exhibited by a subspace such as arbitrary intersection of a subspace is a subspace, union of two subspace is a subspace if and only if one is contained in the other, etc.

Content of the Unit:

Definition of real vector space, examples with real entries, space of real valued functions on a non empty set, Subspace: Definition, examples of subspaces such as lines, plane passing through origin upper triangular, lower triangular, diagonal, symmetric and skew-symmetric matrices as subspaces, solutions of m homogeneous linear equations in n unknowns as a subspace; space of continuous real valued functions on a nonempty set, properties of subspace such as necessary and sufficient condition for a non empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace; union of two subspaces is a subspace if and only if one is a subset of the other, Linear combinations of vectors in a vector space.

Unit III: Basis (15 Lectures)
Learning Outcomes:

1. Define linear span, linear independence and linear dependence of a set of vectors, basis of a vector space.
2. Determine whether a set is linearly dependent or linearly independent.
3. Check whether a given set forms a basis for a given vector space.

Content of the Unit:

Linear span of a non empty subset N of a vector space, N is the generating set of linear span of a non empty subset of a vector space is a subspace of the vector space, Linearly independent / Linearly dependent sets in a vector space, properties Basis of a vector space, Dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any two basis of a vector space have the same number of elements, Quotient space.

Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited, 1991.
2. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India Pvt. Ltd, 2000.
3. David Lay, Linear algebra and its Applications.

Additional Reference Books:

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
4. L. Smith: Linear Algebra, Springer Verlag.
5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.

Assignments:

1. Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1,2), (1,3), (2,2), (2,3), (3,3)$.
2. Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.
3. Verifying whether given $(V, +, .)$ is a vector space with respect to addition $+$ and scalar multiplication.
4. Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3(X)$; $M_3(\mathbb{R})$ etc. Verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.

Practical based on UGMT201:

1. Limit of a function and Sandwich theorem, Continuous and discontinuous functions, Properties of continuous functions.
2. Differentiability, Higher order derivatives, Leibnitz theorem.

3. Mean value theorems and its applications.
4. Extreme values, increasing and decreasing functions
5. Applications of Taylor's theorem and Taylors polynomials.

Practical based on UGMT202:

1. Solving homogeneous system of m equations in n unknowns by elimination for $(m, n) = (1,2), (1,3), (2,2), (2,3), (3,3)$.
2. Solving system $Ax = b$ by Gauss elimination, Solutions of system of linear Equations.
3. Verifying whether given $(V, +, .)$ is a vector space with respect to addition $+$ and scalar multiplication.
4. Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3(X)$; $M_3(\mathbb{R})$ etc. Verifying whether a set is a basis of a vector space. Extending basis of a subspace to a basis of a finite dimensional vector space.

Scheme of Examination

For UGMT101, UGMT102, UGMT201 and UGMT202 (Semester I & II)

A. There will be a **Semester end examination** of 60 marks.

1. Duration: The examinations shall be of 2 Hours duration.

2. Theory Question Paper Pattern:

a) There shall be FOUR questions. The questions first three questions shall be of 15 marks each based on the units I, II, III respectively. The fourth question shall be of 15 marks based on the entire syllabus.

b) All the questions shall be compulsory. The questions shall have internal choices within. Including the choices, the marks for each question shall be 30.

c) The questions may be subdivided into sub-questions and the allocation of marks depends on the weightage of the topic.

B. There will be **Continuous Internal Assessment** of 40 marks.

Paper	20 Marks	10 Marks	10 Marks
Paper I (Calculus)	Unit Test	Assignment	Class Test on Unit II
Paper II (Algebra)	Unit Test	Assignment	Class Test on Unit II

Question paper pattern for Unit Test of 20 marks:

The unit test for 20 marks will be conducted online. There shall be 20 compulsory multiple choice questions with single correct answer, each carrying one mark.

C. Practical Examination

1. There will be semester end practical examination of 100 marks.
2. Duration: The examinations shall be of **3 Hours** duration.

Practical Exam	Viva	Journal	Total
80	10	10	100

Question paper pattern for practical exam of 80 marks:

Part A: Based on Paper I (Total 40 marks)
Section I: Multiple Choice Questions (Total 16 marks, 2 marks each)
Attempt any **8** out of **12**
Section II: Attempt any **THREE** out of **FIVE** (Total 24 marks, 8 marks each)

Part B: Based on Paper II (Total 40 marks)
Section I: Multiple Choice Questions (Total 16 marks, 2 marks each)
Attempt any **8** out of **12**
Section II: Attempt any **THREE** out of **FIVE** (Total 24 marks, 8 marks each)

Each Practical of every course of Semester I and II shall contain 10 problems out of which minimum 05 have to be written in the journal. A student must have a certified journal before appearing for the practical examination.